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Calculation of Reactances for Ring Windings to Toroidal Inductors of Hybrid Induction Machine

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***Abstract*—The linear induction motors of hybrid structure have a series of particularities and their most precise estimation of practical flows and outlets has a high level of complexity, especially due to the different form of inductor. The personal contribution on theoretical ground is the justification of the relations for the calculation of practical reactances and outlets, which were also validated by the experimental results using the method of effects' overlapping. The theoretical and experimental results confirm the accuracy of methods and greatly confirm the proposed methods.**

***Index Terms*—equivalent circuitry, hybrid linear induction machine, magnetic core, slot, magnetization reactance**

I. INTRODUCTION

In this paper, the general expressions for self and mutual inductances of linear motors with hybrid structure were derived by using weights method. The theoretical considerations were carried out for three-phase winding-up in one layer, with even number of poles and one slot per pole and phase, as it is in our practical implementation. The magnetic asymmetries due to the different magnetic coupling of edge phases R and, respectively, S with the mean phase, are taken into account. The self inductance of mean phase T is always greater than of the other phases because it is edged by greater lengths of magnetic circuits. The magnetization reactances are determined by three methods, including the general method used for the induction motors. Experimental testing of these expressions was based on the hybrid engine built in our laboratory.

II. EQUIVALENT CIRCUITS

By analogy with the characteristics of asynchronous and linear rotary induction machine [1], the parameters of the hybrid motor are the resistances and reactances of outlets for the primary winding X_1, R_1 and for the armature board X_2, R_2, the magnetization reactance X_m, and the calculation resistance R_m, which takes into account the lossses of inductor magnetic circuit.

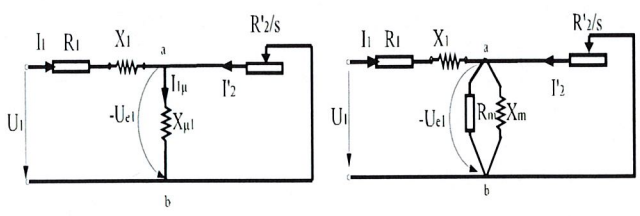


Figure 1. The equivalent circuit of the hybrid induction engine.

In Fig.1, the equivalent electric circuits are indicated for a phase of the hybrid linear machine of induction obtained by neglecting the inductivity of non-magnetic disk in the electric circuits known for rotary machines.

Similar to the asynchronous induction machine functioning in different modes, other parameters of hybrid machine do not remain constant. The total magnetic field flux, which links the magnetic circuitry of the machine, Φ_{11}, can be written as the sum of two components:

$$\Phi_{11} = \Phi_m + \Phi_1 \tag{1}$$

where: Φ_m is the practical flux, including the component that links conductors of secondary circuitry, while Φ_1 is the outlets flux that does not link the secondary, but only the self coils. Each of the fluxes is related to a reactance, which intercedes with operating equations, as well as machine characteristics calculations. Therefore, both magnetization reactances X_m, and outlets reactances X_1 have to be calculated.

III. THE MAGNETIZATION REACTANCE

For rotary asynchronous machines, the magnetization reactance of a distributed winding [2] for the fundamental harmonic has the following expression:

$$X_m = 1.6\, m_1 \frac{f_1}{100}\left(\frac{w_b k_w}{100}\right)^2 \frac{\tau}{100\delta K_C^2}\frac{l}{p}[\Omega] \tag{2}$$

The number of phases is denoted by m_1, the frequency of supply voltage by f_1 (in [Hz]), the number of coils on phase connected in series by w_b, the winding factor of the phase main wave for the inductor winding by k_w, the number of poles pairs by p, the polar step by τ, the length of magnetic gap by δ, the ideal length of the armature by l; with all sizes considered in [cm].

The magnetization reactance [3] of a hybrid engine with linear inductor equipped with ring winding is:

$$X_m = 2\pi f_1 \cdot \mu_0 \cdot \frac{(2w_b)^2 b_d \cdot l}{K_C^2 \cdot \delta} \cdot \frac{p(30p+14)}{3\cdot(6p+1)}[\Omega] \tag{3}$$

where K_C is the Carter factor [4], which is modified in K_C^2 for the bilateral engines; b_d is the length of tooth and l the length of inductor. In order to have an idea concerning the validity of those two formulas, (2) and (3), the following calculation was carried out.

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A critical analysis of the relation (3) was first done. By considering the way that relation was deducted, two main observations can be made. The first one is related to the fact that from the entire area of primary armature traversed by the inductor flux only the teeth area is considered, therefore by neglecting the flux traversing the gap from slot's area. As a result, instead of tooth length, the tooth step τ_d will be taken into consideration, since the bigger gap placed in front of the slot is taken into account, by the Carter's factor. As consequence, relation (3) will be modified as following:

$$X_m = 2\pi f_1 \cdot \mu_0 \cdot \frac{(2w_b)^2 \tau_d \cdot l}{K_C^2 \cdot \delta} \cdot \frac{p(30p+14)}{3 \cdot (6p+1)} [\Omega] \quad (4)$$

where τ_d is the step of slot $\tau_d = b_d + b_c$, and b_c is the length of the inductor's slot.

The second observation is that relation (3) was established for the particular situation of the inductor, built and tried, upon basis of general relation, more precisely, for $2p = 12$.

IV. THE OUTLETS REACTANCES

The armature board is formed of material with significant electrical properties (low resistivity), but without magnetic properties (non-magnetic material, as aluminum, copper), being the basis of secondary currents.

By taking into account of the above mentioned characteristics of the armature and the fact that number of its coils is reduced to 1, it is apparent that the outlets flow of the secondary is extremely low and, consequently, it can be neglected. The outlets flows for the primary armature remain, whose magnetic core is presented in (Fig.2). Within the inductor's core slots, the ring windings are introduced. The ring windings present a series of important advantages, which take into consideration both choosing the polar step, and the arrangement of windings in slots. The connection in series is made easily, on the opposite side of the slots for the inductor's magnetic core. Foe the hybrid engines with toroidal inductor (Fig.3), both known reactances, considered currently for three-phased rotary induction engines, and the particular reactances of the hybrid engines [3] , which have certain effects as result of finite size of armatures and longer gap, will be taken into consideration.

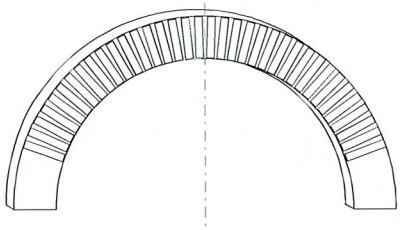


Figure 2. Designed inductor of the hybrid induction engine.

A series of supplementary determined outlets [3] are taken into consideration, specific to hybrid engines (Fig.4), with ring windings: due to lateral sides of teeth Φ_{S2d} and Φ_{S3d}, to lateral areas of the yoke Φ_{S1}, the flow of superior caps Φ_{S21s} and Φ_{S31s} and inferior caps, towards slots side and inferior caps, noted in below figure with Φ_{S2s} and Φ_{S3s}, on lateral areas of the edge teeth. Within the outlets' flow, it is also included that flow corresponding to difference between the magnetic field resultant totally in the gap, including that due to superior harmonics and the magnetic field.

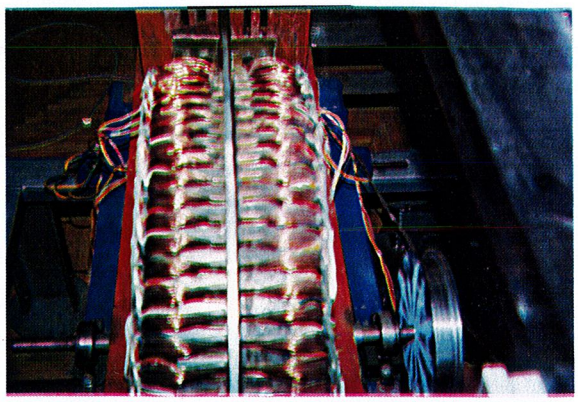


Figure 3. The bilateral inductor of the hybrid engine tested in laboratory.

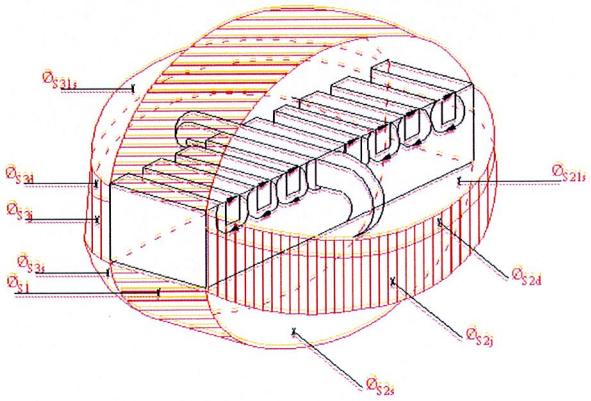


Figure 4. The outlets flows.

In [2] graphical-analytical methods are presented, by means of differential outlets coefficient k_d. From the magneto-motive force analysis of a three-phased winding, displaced in area with angles of 120^0 and of 240^0, supplied by time phase-shifted sinusoidal currents with the same angles of 120^0 and 240^0, for each winding and phase the equations can be written. If the origin is set as phase R, then, the system (5) gives the values of magneto-motive forces on the three phases:

$$\begin{aligned}
F_{R[\alpha,t]} &= \left[F_{f1}\cos\alpha + F_{f3}\cos 3\alpha + F_{f5}\cos 5\alpha + \ldots + F_{f\nu}\cos\nu\alpha\right]\sin\omega t \\
F_{S[\alpha,t]} &= \left[F_{f1}\cos\left(\alpha-\frac{2\pi}{3}\right) + F_{f3}\cos 3\left(\alpha-\frac{2\pi}{3}\right) + F_{f5}\cos 5\left(\alpha-\frac{2\pi}{3}\right) + \ldots + F_{f\nu}\cos\nu\left(\alpha-\frac{2\pi}{3}\right)\right]\sin\left(\omega t-\frac{2\pi}{3}\right) \\
F_{T[\alpha,t]} &= \left[F_{f1}\cos\left(\alpha-\frac{4\pi}{3}\right) + F_{f3}\cos 3\left(\alpha-\frac{4\pi}{3}\right) + F_{f5}\cos 5\left(\alpha-\frac{4\pi}{3}\right) + \ldots + F_{f\nu}\cos\nu\left(\alpha-\frac{4\pi}{3}\right)\right]\sin\left(\omega t-\frac{4\pi}{3}\right)
\end{aligned} \quad (5)$$

$$F_{[t,\alpha]} = F_1\sin(\omega t-\alpha) + F_5\sin(\omega t+5\alpha) + F_7\sin(\omega t-7\alpha) + F_{11}\sin(\omega t+11\alpha) + F_{13}\sin(\omega t-13\alpha) + \ldots + F_\nu\sin(\omega t\pm\nu t) \quad (6)$$

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In (6), the relation of resultant magnetomotive force of the three phases is given. All harmonics of order $\nu = 6k - 1$, i.e. $\nu = 5, 11, 17, \ldots$, are rotating towards left in opposite sense of fundamental wave. All harmonics of order $\nu = 6k + 1$, i.e $\nu = 1, 7, 13, \ldots$, are moved towards right, meaning in the moving sense of fundamental wave and of the armature. In the above relations, notations are:

$$\omega t = \frac{2\pi}{T} \quad ; \quad a = \frac{2\pi}{T_e}$$

where: T and T_e are the period of wave, depending on time and area.

Next, it is preferred that relations of inductive reactances ($X = \omega L$) to be expressed depending upon frequency and inductivity, by using magnetic resistance R_m, or more obvious, by its inverse, magnetic permeance Λ. For a coil (with total number w_e of turns on slot) placed within a slot, the inductance and conducting flow w_x are:

$$L = \frac{\sum w_x \Phi_x}{i}, \quad \Phi_x = \frac{w_x i}{R_x}$$

It is known that permeance is:

$$\Lambda = \frac{1}{R_m} = \frac{1}{\frac{l}{\mu_x A}} = \mu_x \frac{A}{l} \tag{7}$$

The permeance (7) is a measure proportional with the magnetic properties of the environment, by which the magnetic flow is closed and depends upon the geometrical sizes of the closing route. If the outlets flow traverses an air shell, it might be considered that $\mu_x = \mu_0$. Therefore, if appreciations are done to the same magnetic material, a specific permeance can be defined, as depending upon the geometrical dimensions. Continuously, specific measures are considered for work, noted with λ_x, which represent the permeance coefficient of the outlets flow on length unit of a coil. If the outlets flow links through the air an ordinary number of coils w_x, each traversed by current i, then their outlets flow and corresponding inductance are given by:

$$\Phi_x = w_x \Lambda_x i = \mu_0 w_x \lambda'_x l_x i, \quad L = \mu_0 \sum w_x^2 \lambda'_x l_x$$

The outlets phase inductance [2] for a winding, placed within q slots, with p pairs of poles, with a number w of coils, is:

$$L = \mu_0 \cdot \frac{w^2}{p \cdot q} \cdot \sum \lambda_x \cdot l_x \tag{8}$$

where: l_x – is the length of a turn;

λ_x – represents the specific permeance equivalent of the outlets flow, that links the total number of conductors placed in slots. The outlets reactance of the entire phase, with the coils connected in series, is:

$$X_1 = \omega \cdot L_\sigma = 2\pi f \mu_0 \frac{\omega^2}{pq} \sum \lambda_x \cdot l_x \tag{9}$$

The expression resulted is valid for any type of windings, with the condition of a convenient calculation for the sum $\sum \lambda_x \cdot l_x$.

Appreciations over the size of specific permeances' values, for a winding in one layer, with pole slot and phase ($q = 1$) are next taken, where are considered to be minimal (reduced) the two known phenomena, which usually accompany the starting of hybrid engine. First of those is the influence of secondary current repression, physical phenomena of greater intensity on starting, when the secondary frequency is close to primary frequency value ($f_2 = s f_1$), and therefore, the secondary reactance increases. The second characteristic phenomenon, when the starting sink currents are significant, but only within starting process, is the increase of saturation for ferromagnetic circuitry, accompanied by increase of magnetic circuitry resistance, by which both lines of transversal flow on slot are closed, and also the lines of field produced of superior harmonics of the magnetic field of gap. The relative step of the winding is considered to be unitary ($\beta = 1$)..

V. THE SLOT PERMEANCE

In Fig.5, the outlets flow Φ_l is presented on the height of a semi-closed slot, for a winding with one layer. The resistance of iron is neglected.

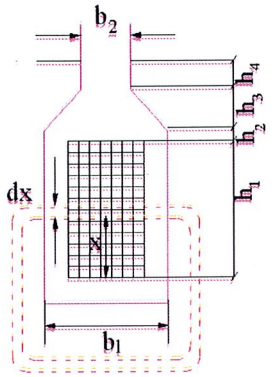


Figure 5. The outlets flow on slot.

From this reason, assuming that lines of induction pass all over the slot, straight-line and parallel one towards each other, it might be assumed that for each elementary tube, its length that determines the self-inductance, is equal to the length of slot towards that height form datum line.

For the outlets flows that have a total number w_e of conductors on slot, the permeance of tube on length slot unit λ_x is determined, and also the relation (7), as report between the thickness of tube and the slot height considered, reported to the mean width of the slot towards that height. For the slot of Fig.4, the permeance is:

- for the part of height h_2:

$$\lambda_{e2} = \frac{h_2}{b_1} \tag{10}$$

- for the part of height h_3:

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$$\lambda_{e3} = \frac{h_3}{\frac{1}{2}(b_1 + b_2)} = \frac{2h_3}{b_1 + b_2} \tag{11}$$

- for the part of height h_4:

$$\lambda_{e4} = \frac{h_4}{b_2} \tag{12}$$

The equivalent permeance of the outlets flow that will meet all these w_e conductors of slot, can be determined by means of outlets flow, which links only a part of conductors on the direction of slot height h_1. An elementary tube of width dx that exists on height x towards the slot's base has an elementary permeance:

$$\lambda'_x = \frac{dx}{b_1} \tag{13}$$

This tube surrounds a number of conductors of the slot:

$$w_x = w_e \frac{x}{h_1} \tag{14}$$

For the length unit of the inductor:

$$w_x^2 \lambda'_x = \frac{w_e^2}{h_1^2 b_1} x^2 dx \tag{15}$$

The total flow for all the conductors that occupy the height h_1 is proportional with:

$$\sum_0^{h_1} w_x^2 \lambda_x = \int_0^{h_1} \frac{w_e^2}{h_1^2 b_1} dx = w_e^2 \frac{h_1}{3b_1} = w_e^2 \lambda_{e1} \tag{16}$$

The total equivalent permeance of the entire slot is:

$$\lambda_e = \lambda_{e1} + \lambda_{e2} + \lambda_{e3} + \lambda_{e4} = \frac{h_1}{3b_1} + \frac{h_2}{b_1} + \frac{2h_3}{b_1 + b_2} + \frac{h_4}{b_2} \tag{17}$$

VI. PERMEANCE THE FRONT COILS OF THE RING

To rotary induction machines [2], in situation of coils that form windings on a single layer, the permeance of frontal parts is:

$$\Lambda_f = l_f \lambda_f = 0{,}67 l_f - 0{,}43\tau \tag{18}$$

Considering that hybrid machine with toroidal inductor uses the ring windings, the classical calculation formulas of this permeance cannot be applied. The frontal parts permeance will be taken into consideration in the near future.

VII. INDUCTIVE DIFFERENTIAL REACTANCE ON SLOTS

For a hybrid bilateral engine, on mean radius R, the expression of inductive reactance X_d of differential outlets can be written, of a winding phase, if the mutual induction of the superior field harmonics to the other phases windings are taken into consideration, by relation:

$$X_d = \sum_{\nu>1} x_\nu = 2mf \frac{\mu_0 Dl}{K_c k_\mu \delta} \cdot \frac{w^2}{p^2} \sum_{\nu>1} \frac{k_{b\nu}^2}{\nu^2} \tag{19}$$

where the sum is related to all harmonics, except the fundamental harmonic ($\nu = 1$).

The significance of notations of relation (19) is:

- D diameter of inductor magnetic circuitry;
- l active length of iron (inductor);
- K_C Carter's factor;
- k_μ coefficient of iron saturation;
- δ measure of mechanic gap;
- $k_{b\nu}$ factor of winding resulted for the harmonic of order ν.

The coefficient of differential outlets is defined, as being the report between inductive reactance on differential outlets on phase, to inductive reactance of fundamental harmonic of the gap field:

$$k_d = \frac{X_d}{X_m} \tag{20}$$

Then number of windings of pole end phase is q. Its measure for the three-phased windings is given under graphical form also, depending upon q (Fig..6) or depending upon q and that relative step of the winding β (Fig. 7), as mentioned below:

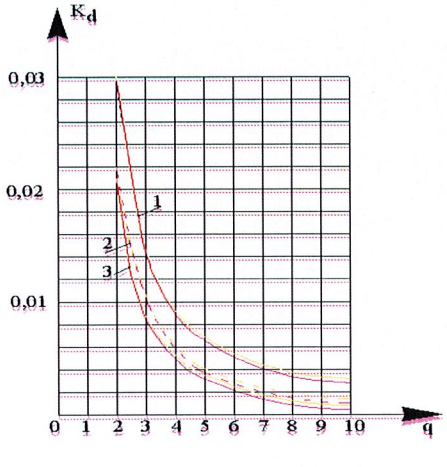


Figure 6. Values of kd for the three-phased windings depending upon q = z/3·2p.

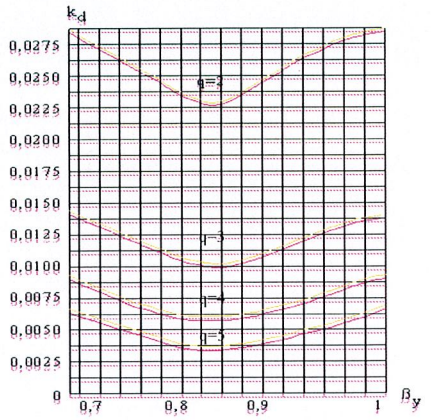


Figure 7. Curves of differential dispersion coefficient.

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With relations (2), (19)-(20), the differential reactance can be written, depending upon the corresponding permeance:

$$X_d = 4\pi f\ \mu_0 \frac{lw^2}{pq} \lambda_d \qquad (21)$$

From the above relation, one might deduce the permeance of differential outlets flow, given by:

$$\lambda_d = \frac{mqk_{b1}^2 \tau}{\pi^2 k_\delta k_\mu \delta} k_d \qquad (22)$$

The self-induction reactance of the winding, taken into consideration the mutual induction of the other phases for the harmonic of ν level, is the following:

$$X_\nu = 2mf \frac{\mu_0 Dl}{K_c k_\mu \delta} \frac{w^2}{\nu^2 p^2} k_\nu^2$$

The inductive reactance corresponding to harmonics is proportional with the frequency and the square of coils' number, but fast diminishes once the harmonic ν^2 increases.

VIII. Results

The magnetization reactances were determined for a bilateral three-phased hybrid engine, with the following characteristics:

U = 400V-coltage on outside lines
f = 50Hz – frequency of supply voltage
b_{dmed} = 16 [mm] – the width of a tooth;
b_{d0} = 70 [mm] - the width of edge tooth;
b_c = 4 [mm] – the width of slot;
h_c = 45 [mm] – the height of slot;
h_j = 25 [mm] – the height of yoke;
l_i = 120 [mm] – the width of inductor;
L = 860 [mm] –the length of inductor;
δ = 14 [mm] – the gap;
N = 90 number of turns – the number of coil windings.

With relation (2) for all rotary induction engines, it was determined that X_m = 22,59 Ω, and for that with hybrid engines on linear inductors (3) resulted that X_m = 18 Ω, when the justified relation (4) was used, resulting for the magnetization reactance X_m = 22,51 Ω.

XI. CONCLUSIONS

The theoretical considerations were carried out for three-phase winding-up in one layer, with even number of poles and one slot per pole and phase, as it is in our practical implementation. The magnetic asymmetries due to the different magnetic coupling of edge phases R and, respectively, S with the mean phase, are taken into account. The self inductance of mean phase T is always greater than of the other phases because it is edged by greater lengths of magnetic circuits. The magnetization reactances are determined by three methods, including the general method used for the induction motors. Experimental testing of these expressions was based on the hybrid engine built in our laboratory.

The method of overlapping the effect is considered by authors more rigorous, because it takes into account the entire assembly of phenomena and the flux parts that close the field lines through magnetic circuits. In order to understand this method an example was taken and finally, applied to the concrete situation for which the engine was designed and built. The hybrid engine (Fig.8) was built and tested within the laboratory.

The experimental results have confirmed the theoretical calculations.

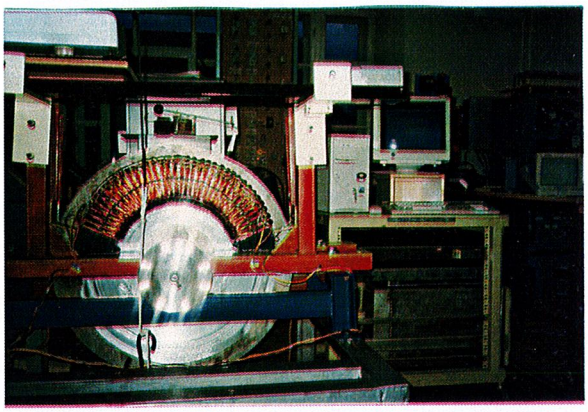


Figure 8. The hybrid engine built and tested within the laboratory.

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